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ANOMALIES OF THE PROPAGATION OF LONGITUDINAL SOUND IN A LAYER OF A DISSIPATIVE MEDIUM

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Normal propagation of continuous and pulse longitudinal acoustic waves through the gap between two solid half-spaces, which is filled with a dissipative medium, has been considered theoretically. A strong dependence of the transmission and reflection coefficients and of their phases on the coefficient of damping of longitudinal sound in the medium of the gap and on its dimensionless phase thickness has been shown. The shapes of the reflected acoustic pulsed signal and of the signal transmitted by the gap have been calculated using software. The results obtained are used in investigation of the acoustic properties of viscous fluids. Application of the transformed spectrum of the signals to development of radiation devices and processing of information is discussed.

Introduction. The propagation of bulk acoustic waves without absorption in stratified media has been considered rather thoroughly [1], although analytical solutions have been found for a three-layered medium only in the case where the incidence of the wave is normal to the interfaces. If it is required by practical applications, oblique incidence of the wave on the boundaries of a stratified structure must be calculated on a computer. In actual experiments and engineering applications, one often deals with frequency-broadband signals of finite, very short, length rather than with continuous acoustic vibrations. In solids and most liquids, except for the resonances of interaction between the acoustic waves and other elementary excitations of a substance (e.g., spin waves [2]), dispersion of the velocity of sound is absent up to high frequencies at which its excitation in the form of directional radiation has not been realized until the present time. Nonetheless, there exist fluids with considerable viscosity or mixtures of substances in chemical reaction between them in which dispersion of sound occurs.

If the dispersion of sound is absent in the materials composing the stratified structure, all the frequency components of the pulsed signal are reflected from the interface according to classical Fresnel formulas, which are frequency-independent and, consequently, the spectrum of transformed signals does not change. In the case under consideration (Fig. 1), by virtue of the existence of sound dispersion in a strongly dissipative medium (SDM), the frequency components of the pulsed signal in its incidence *I* on the interface are transformed differently and, accordingly, the spectrum of the signal reflected from the interface *II* and transmitted by it *III* changes [3]. In this case, depending on the number of wavelength quarters present on the layer thickness, the frequency dependence of the coefficients of reflection and transmission arises due to the interference of the incident, reflected, and transmitted waves.

Theory. Let a continuous harmonic longitudinal wave, which is partially reflected and finds its way to a layer of a strongly dissipative medium and then to solid half-space 2, be incident from solid half-space 1 to the layer of a strongly dissipative medium (Fig. 1). The wave equations for the longitudinal wave in each material of the stratified structure are written as follows [1, 4]:

$$\rho_1 \ddot{u}_x = c_1 u_{x,xx} , \quad \rho \ddot{u}_x = c u_{x,xx} + b u_{x,xxt} , \quad \rho_2 \ddot{u}_x = c_2 u_{x,xx} , \tag{1}$$

where $u_{x,xx} = \partial^2 u_x / \partial x^2$, $u_{x,xxt} = \partial^3 u_x / \partial^2 x \partial t$, and b is the parameter of dissipative loss determined according to the relation [1]

$$b = \frac{4}{3}\eta + \xi + \chi \left(c_v^{-1} + c_p^{-1}\right) . \tag{2}$$

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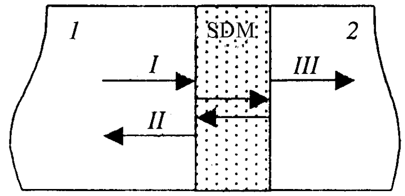

Fig. 1. Propagation of the longitudinal wave in the stratified structure.

We find solutions for the longitudinal wave in the half-spaces ($x < 0$, $x > d$) and the layer ($0 < x < d$) in the form [4]

$$u_{1x} = A_1 \exp [i (k_1 x - \omega t)] + B_1 \exp [i (-k_1 x - \omega t)] , \quad u_x = A \exp [i (kx - \omega t)] + B \exp [i (-kx - \omega t)] ,$$

$$u_{2x} = A_2 \exp [i (k_2 x - \omega t)] , \tag{3}$$

where $k_1 = \omega/s_1$ and $k_2 = \omega/s_2$ are the wave numbers. The law of sound dispersion in the strongly dissipative medium is written as follows [1]:

$$k^2 = (1 - i\omega b/c)^{-1} \omega^2/s_0^2 , \tag{4}$$

where $k = k' + ik''$ (k' and k'' are its real and imaginary parts) and $s_0 = (c/\rho)^{1/2}$.

Relations for the phase and group velocities follow from (4):

$$v_{\rm ph} = s_0 [2 (1 + x^2)]^{1/2} / [2 (1 + x^2)]^{1/2} + 1]^{1/2} , \tag{5}$$

$$v_{\rm gr} = \frac{2ks_0^2 (1 + x^2) [(1 + x^2)^{1/2} + 1]}{(1 + x^2)^2 \left\{x\omega_{\rm c} + 2s_0 k^2 [(1 + x^2)^{1/2} + 1]/\omega_{\rm c}\right\} - (1 + x^2)^{1/2} (2x\omega_{\rm c} + 1/2) + x\omega_{\rm c}} , \tag{6}$$

where $x = \omega/\omega_{\rm c}$ and $\omega_{\rm c} = c/b = \rho s_0^2/b$. Analysis of (5) and (6) shows that with increase in the frequency we have ($\omega \to \infty$) $v_{\rm ph} \to \infty$ and $v_{\rm gr} \to 0$.

For k' and k'' parts of the wave number we find the following expressions:

$$k'^2 = k_0^2 \frac{(1 + x^2)^{1/2} + 1}{2 (1 + x^2)} , \quad k''^2 = k_0^2 \frac{x^2}{2 (1 + x^2) [(1 + x^2)^{1/2} + 1]} , \quad k_0 = \omega/s_0 . \tag{7}$$

If the real part of the wave number decreases from $k_0 = \omega/s_0$ to zero, the imaginary part has a maximum at the frequency $\omega \approx \omega_{\rm c}$, and when $x \to 0$ and $x \to \infty$, $k'' \to 0$. Their ratio is always $k''/k' < 1$ and is a slowly increasing function of the argument x. Since we consider sound of weak intensity, first, it will damp exponentially in the absorbing layer and the thus-caused slight heating of the medium will not lead to the distortion of the wave characteristics; second, in Eqs. (1), it suffices to restrict ourselves to the linear terms only, since the contribution of the square terms and of the terms of higher order will be negligibly small compared to the first ones. If as the layer of a strongly dissipative medium we use a rheological fluid whose viscosity changes by a factor 10^6 or more under the effect of the electric or magnetic field [5], or a magnetoacoustic material where the absorption of ultrasound greatly increases with distance to the point of orientation phase transition along the outer magnetic field [2], then, for such materials, there is a unique possibility of electronic control over the spectral characteristics of the signal (amplitude, phase, duration, frequency band); this can be employed in developing devices of signal transformation in processing of information. Another important application of the considered effect can be study of the characteristics of friction pairs (coefficient of friction, friction force and its momentum, wear) [6] which are usually present in one form or another in different mechanisms, and knowledge of tribological characteristics determines the operating characteristics, reliability, and longevity of machines.

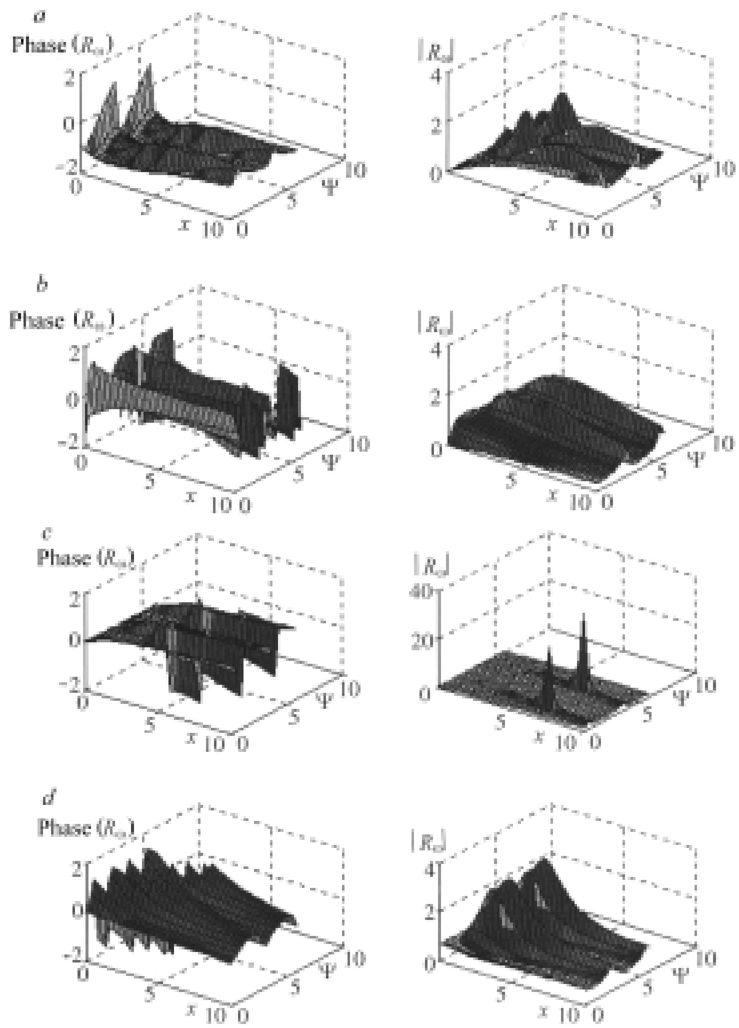

Fig. 2. Phase and modulus of the reflection coefficient for the stratified structure: a) plastic–ER–plastic; b) aluminum–ER–aluminum; c) aluminum–ER–plastic; d) plastic–ER–aluminum; for aluminum: Z = 17.33·10^6 kg/(m^2·sec) and s = 6.42 km/sec; for ER — 3.25·10^6 and 2.68; for plastic — 3.1·10^6 and 2.7. ω_c = $2\pi\cdot 10^7$ Hz; $\Psi = k'd$.

At present, there are rather numerous methods for measuring the viscosity of liquid media [7], but all these methods require direct mechanical contact between the measuring device and the fluid; they are long-lasting, restore the true value of viscosity from the nonlinear calibration curve of an indirectly measured parameter, and are suitable for a narrow range of substances. The technique suggested is instantaneous and is suitable not only for measuring the viscosity of a wide class of liquid media but also for recording the internal friction in solids.

The boundary conditions represent the continuity of elastic displacements and stresses on the interface and have the form

$$u_{1x} = u_x\,, \quad c_1 u_{1x,x} = c u_{x,x} + b u_{x,xt} \quad \text{at} \quad x = 0\,; \quad u_x = u_{2x}\,, \quad c u_{x,x} + b u_{x,xt} = c_2 u_{2x,x} \quad \text{at} \quad x = d\,. \tag{8}$$

The solutions (3) satisfy the corresponding wave equations and, being substituted into (8), give the system of linear equations for determination of the coefficients of reflection $R_\omega = B_1/A_1$ and transmission $T_\omega = A_2/A_1$. The reflection coefficient is determined from the relation

$$R_\omega = \frac{A - B}{A + B}\,, \tag{9}$$

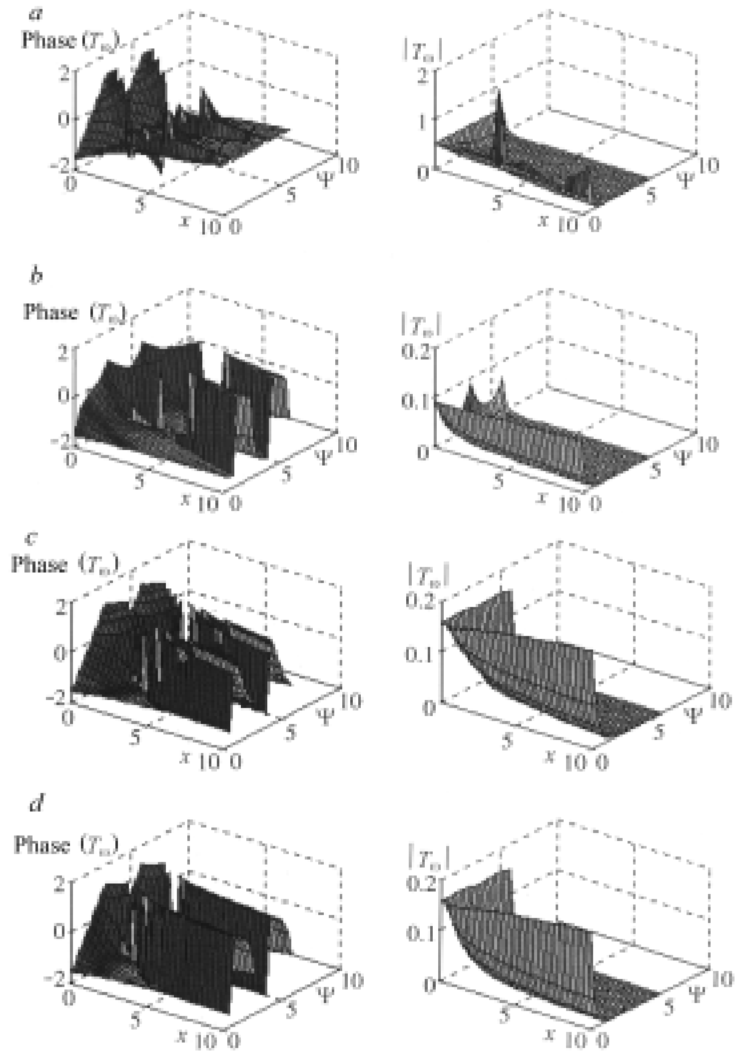

Fig. 3. Phase and modulus of the transmission coefficient for the considered stratified structures. For the notation see Fig. 2.

where

$$A = Z_1 Z_2 X_- + iZ\,(Z_1 - Z_2)(1 - ix)(i\gamma - \beta)X_+;\; B = Z^2\,(1 - ix)^2 (i\gamma - \beta)X_-;$$

$$X_+ = 2\cos(\alpha)\cosh(\beta) - 2i\sin(\alpha)\sinh(\beta);\; X_- = 2\cos(\alpha)\sinh(\beta) - 2i\sin(\alpha)\cosh(\beta);$$

$$Z = \rho s_0;\; \alpha = \omega/(\sqrt{2}s_0);\; \beta = k''d;\quad \gamma = k'd.$$

It is seen from Eq. (9) that the expression for the coefficient of reflection of a continuous longitudinal wave from the dissipative layer is rather cumbersome. Because of this, the frequency dependence of the phase and the modulus of the reflection coefficient (Fig. 2) for the stratified structure having a layer of epoxy resin (ER) was calculated on a computer by numerical methods. The reflection coefficient is $R_\omega = (Z_1 - Z_2)/(Z_1 + Z_2)$ when $\omega \to 0$ and $R_\omega \to 1$ when $\omega \to \infty$. With change in the frequency, when the length $n\lambda/4$ (n is an integer) is present on the layer thickness, we have the extrema of R_ω, i.e., oscillations of the coefficient of reflection of the longitudinal wave arise.

We obtain the following relation for the coefficient of transmission T_ω of the longitudinal wave:

$$T_\omega = \frac{-4iZ_1 Z\,(1 - ix)\,(i\gamma - \beta)}{A + B}\exp(-ik_2 d)\,. \tag{10}$$

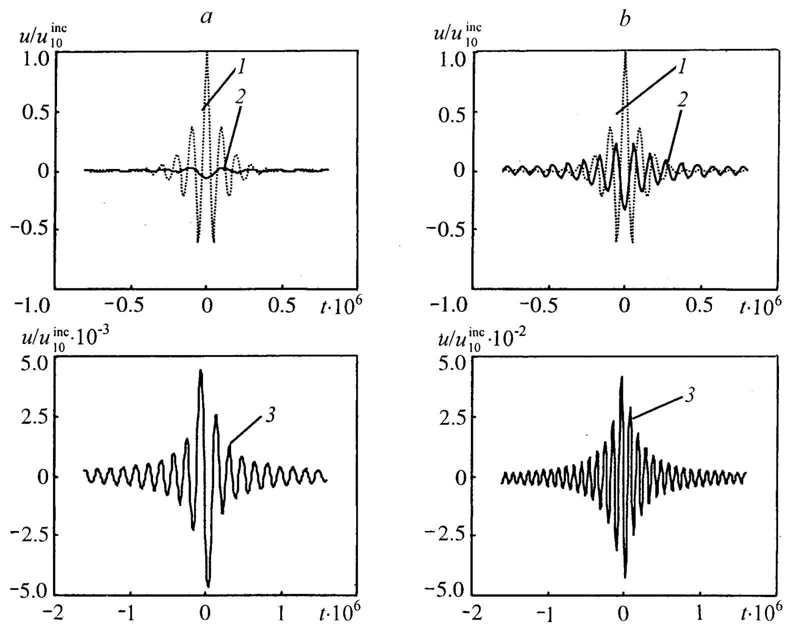

Fig. 4. Reflected and transmitted signals for the plastic–ER–aluminum structure [a) ω/ω_c = 0.1; b) 1; the thickness of the ER layer is 2 mm]: 1) u_1^{inc}/u_{10}^{inc}; 2) u^{ref}/u_{10}^{inc}; 3) u^{tran}/u_{10}^{inc}.

Figure 3 gives the frequency dependences of the phase and the modulus of the transmission coefficient. With change in the frequency, when the length $n\lambda/4$ is present on the layer thickness, the arising minima (maxima) of T_ω correspond to the maxima (minima) of R_ω.

An actual pulsed acoustic signal can be represented by the expression

$$u_1^{inc}(x=0, t) = u_{10}^{inc} \exp\left(-\Gamma \frac{|t|}{T}\right) \exp\left(i2\pi \frac{t}{T}\right) \left[\theta\left(t - \frac{\tau}{2}\right) - \theta\left(t + \frac{\tau}{2}\right)\right], \tag{11}$$

where the envelope of the pulsed signal is related to the quality Q by the expression $\Gamma = \pi/Q$; $T = 2\pi/\omega_0$; $\tau = nT$, n is equal to the number of periods of the radiated pulse.

Proceeding from the presented relations for R_ω (9) and T_ω (10) and using direct and inverse Fourier transforms

$$F(\omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} u_1^{inc}(x=0, t) \exp(i\omega t)\, dt, \tag{12}$$

$$u_1^{ref}(x=0, t) = \int_{-\infty}^{+\infty} F(\omega) R_\omega \exp(-i\omega t)\, d\omega, \tag{13}$$

$$u_2^{tran}(x=d, t) = \int_{-\infty}^{+\infty} F(\omega) T_\omega \exp(-i\omega t)\, d\omega, \tag{14}$$

we calculated the shape of the reflected and transmitted signals in the above-mentioned stratified structures at $n = 6$ and $\Gamma = 1$ with the aid of the computer.

From the results of the calculations shown in Fig. 4, we see a strong dependence of the amplitude and the phase of the reflected and transmitted signals on the frequency ω_0 of the fundamental harmonic of the pulsed signal.

The phase of the reflected pulse is understood in a more general way than in the case of continuous vibrations, namely: as the value of the displacement of the intersection of the transmitted pulse and the time axis relative to the intersection of the radiated pulse and this axis. The transformation (12) applied to the radiated pulse $u_1^{\mathrm{inc}}(x, t)$, the reflected pulse $u_1^{\mathrm{ref}}(x, t)$, and that transmitted by the layer $u_1^{\mathrm{tran}}(x, t)$ gives their spectra which differ from each other [8, 9].

CONCLUSIONS

The developed software allows one to elucidate the distinctive features of the reflection and transmission of radiated signals of any shape. Analytical calculations are partially possible for the simplest shapes of them (e.g., for a rectangular signal or for several periods of a sinusoidal signal, which are impracticable), but the nontrivial frequency-dependent form of R_ω and T_ω makes determination of the spectrum and shape of the reflected or transmitted signals difficult or even impossible.

The state of a strongly dissipative medium qualitatively affects the reflection and transmission coefficients and the phase of both continuous and pulsed acoustic signals. Allowing for the fact that phase measurements are much more accurate than amplitude ones [10], from them one can judge the absorption of sound in the strongly dissipative medium and, making use of the method of inverse problems, restore the time dependence of the viscosity of the prepared substance, which changes due to its restructurization or temperature variations. From the dependences obtained one can judge the readiness of one technological product or another for use [11].

Continuous monitoring of the quality and structure of coatings in deposition of them on substrates of materials is of importance in modern electronic and engineering production. The suggested technique of ultrasonic phase-time measurements allows diagnostics of the fine structure of substances experiencing physicochemical transformations due to the technological processes of molecular and laser epitaxy, electro- and photolithography, electrochemistry, plasma- and vacuum deposition, and brazing. Here, in most of the enumerated cases one observes strong local heating and phase, aggregate, and chemical transformations in some regions of the product which, under these conditions, are strongly dissipative media in their properties, since changes in the density and the elastic moduli and an increase in the absorption of ultrasonic vibrations arise in them. As a result, subsequently one will manage to relate the coefficient of reflection of the longitudinal wave and its phase and spectrum to the characteristics of the state of the coating and its adhesion resistance and, owing to this, it will become possible to flexibly control the synthesis of coating materials with specified optimum properties with reduction of energy consumption during the technological process and increase in their service life.

NOTATION

ρ_1 and ρ_2, densities of the materials of solid half-spaces 1 and 2, kg/m^3; ρ, density of the layer of a strongly dissipative medium, kg/m^3; u_x, component of the elastic displacement in the longitudinal wave (i.e., acoustic signal propagating in the layer of a strongly dissipative medium), m; c_1 and c_2, elastic moduli for solid half-spaces 1 and 2, J/m^3; c, elastic modulus for the layer of a strongly dissipative medium, J/m^3; η, shear viscosity, Pa·sec; ξ, bulk viscosity, Pa·sec; χ, thermal conductivity, W·m·sec/K; c_p and c_v, heat capacities of the medium at constant pressure and volume, J/(kg·K); x, coordinate along the abscissa axis, m; λ, wavelength, m; d, thickness of the layer of an strongly dissipative medium, m; u_{1x} and u_{2x}, acoustic signals propagating in media 1 and 2, m; A_1 and A_2, amplitudes of the acoustic signal propagating in media 1 and 2 in the forward direction, m; B_1, amplitude of the acoustic signal propagating in medium 1 in the backward direction, m; A and B, amplitudes of the acoustic signal propagating in the layer of a strongly dissipative medium in the forward and backward directions, m; ω, cyclic frequency, Hz; k_1 and k_2, wave numbers for media 1 and 2, m^{-1}; s_1 and s_2, velocities of longitudinal sound in media 1 and 2, m/sec; k, complex wave number for the layer of a strongly dissipative medium, m^{-1}; s_0, velocity of longitudinal sound in the layer of a strongly dissipative medium in the absence of dissipation (at $\omega = 0$), m/sec; v_{ph}, phase velocity, m/sec; v_{gr}, group velocity, m/sec; ω_c, characteristic frequency of the strongly dissipative medium, Hz; Z_1 and Z_2, acoustic impedances of media 1 and 2 for longitudinal sound, kg/(m^2·sec); Z, acoustic impedance for the layer of a strongly dissipative medium for longitudinal sound, kg/(m^2·sec); u_1^{inc}, radiated longitudinal acoustic signal, m; u_{10}^{inc}, amplitude of the radiated acoustic signal, m; Γ, dimensionless parameter determining the envelope of the incident acoustic pulsed signal; T, pulse period, sec; ω_0, frequency of the fundamental harmonic of the signal, Hz; θ, Heaviside function; Q, acoustic-pulse quality; τ, acoustic-pulse duration, sec; t, time, sec; u_1^{ref}, acoustic signal reflected from the layer of a strongly dissipative medium to medium 1, m; u_2^{tran}, acoustic signal transmitted by the layer of a strongly dissipative medium to medium 2, m. Sub- and superscripts: 1 and 2, refer to media 1 and 2, respectively; x, longitudinal displacement (coordinate, denotes the abscissa axis); inc, incident; ref, reflected; tran, transmitted; +, positive direction; –, negative direction; ω, refers to frequency, frequency-dependent; gr, group; ph, phase; c, characteristic (as applied to the characteristic frequency of the interface).

REFERENCES

1. M. B. Vinogradova, O. V. Rudenko, and A. P. Sukhorukov, *Theory of Waves* [in Russian], Moscow (1990).
2. Yu. A. Kuzavko and V. G. Shavrov, *Akust. Zh.*, **39**, Issue 6, 1088–1092 (1993).
3. D. A. Kostyuk and Yu. A. Kuzavko, *Pis'ma Zh. Tekh. Fiz.*, **27**, Issue 23, 31–40 (2001).
4. E. Dieulesaint and D. Royer, *Ondes Elastiques dans les Solides* [Russian translation], Moscow (1982).
5. Z. P. Shul'man and V. I. Baikov, *Rheodynamics and Heat and Mass Transfer in Film Flows* [in Russian], Minsk (1979).
6. D. A. Kostyuk (Kostiuk) and Yu. A. Kuzavko, in: *Proc. Int. Conf. on Neutral Networks and Artificial Intelligence*, Brest (1999), pp. 183–188.
7. P Profos (eds.), *Handbuch der industriallen Messtechnik* [Russian translation], Vol. 2, Moscow (1990).
8. D. A. Kostyuk and Yu. A. Kuzavko, in: *Proc. 2nd. Int. Sci.-Eng. Conf. "Ultrasound Techniques and Technology"* [in Russian], Minsk (1999), pp. 140–142.
9. D. A. Kostyuk and Yu. A. Kuzavko, *Vestn. Brestskaga Univ.* [in Belorussian], No. 4, 56–69 (2000).
10. P. V. Novitskii, *Principles of Information Theory of Measuring Devices* [in Russian], Leningrad (1968).
11. D. A. Kostyuk, *Vestn. BGTU, Fiz., Mat., Khim.*, No. 5, 31–34 (2001).